Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

1. The binomial expansion of

$$(3 + kx)^{-2} \qquad |kx| < 3$$

where k is a non-zero constant, may be written in the form

$$A + Bx + Cx^2 + Dx^3 + \ldots$$

where A, B, C and D are constants.

(a) Find the value of A

(1)

Given that $C = 3B$

(b) show that

$$k^2 + 6k = 0$$

(3)

(c) Hence (i) find the value of k

(ii) find the value of D

(3)

(a) $A = (3)^{-2} = \frac{1}{9}$ B1

(b) Get the formula for the Binomial Expansion from the formula booklet:

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$, Range of validity $|x|<1$

We see that the given expansion is in form $(a+bx)^n$, we need to manipulate to get $(1+ax)^n$.

$$(3+kx)^{-2} = \frac{1}{9}\left(1+\frac{k}{3}x\right)^{-2}$$

Substitute into the formula:

$$\frac{1}{9}\left(1+\frac{k}{3}x\right)^{-2} = \frac{1}{9}\left(1+\left(\frac{k}{3}x\right)(-2) + \frac{(-2)(-3)}{2!}\left(\frac{k}{3}x\right)^2 + \ldots\right)$$ we only need up to k^2 $\therefore$ don't expand further

B1

$$= \frac{1}{9}\left(1 - \frac{2}{3}kx + \frac{6}{2}\cdot\left(\frac{k^2}{9}x^2\right) + \ldots\right)$$

$$= \frac{1}{9}\left(1 - \frac{2k}{3}x + \frac{k^2x^2}{3} + \ldots\right)$$

$$= \frac{1}{9} - \frac{2}{27}kx + \frac{1}{27}k^2x^2 + \ldots$$

The coefficient of x^2 is equal to $3\cdot$ coefficient of x (given)

$$\Rightarrow 3\left(-\frac{2}{\cancel{27}}k\right) = \frac{1}{\cancel{27}}k^2 \quad \text{M1}$$

$$-6k = k^2$$

A1 $\Rightarrow \boxed{0 = k^2 + 6k}$ hence shown

Question 1 continued

(c) i. Solve $0 = k^2 + 6k$:

$$0 = k(k+6)$$

$$k = 0 \text{ or } k = -6$$

k is a non-zero constant $\therefore k = -6$ B1

ii. From the formula the coefficient of x^3:

$$\frac{1}{9}\left(\frac{(-2)(-3)(-4)}{3!}\left(\frac{k}{3}\right)^3\right)$$

$$= \frac{1}{9}\left(\frac{\cancel{-24}^{4}}{\cancel{6}_{1}} \cdot \left(\frac{-6}{3}\right)^3\right)$$

$$= \frac{1}{9} \cdot (-4) \cdot (-8)$$

$$= \frac{32}{9}$$ M1A1

(Total for Question 1 is 7 marks)

2. (a) Express $\dfrac{1}{(1+3x)(1-x)}$ in partial fractions.

(3)

(b) Hence find the solution of the differential equation

$$(1+3x)(1-x)\frac{dy}{dx} = \tan y \qquad -\frac{1}{3} < x \leqslant \frac{1}{2}$$

for which $x = \dfrac{1}{2}$ when $y = \dfrac{\pi}{2}$

Give your answer in the form $\sin^n y = f(x)$ where n is an integer to be found.

(6)

(a) We are asked to do Partial Fractions

$$\frac{1}{(1+3x)(1-x)} \equiv \underbrace{\frac{A}{(1+3x)} + \frac{B}{(1-x)}}$$

Manipulate this to get common denominator

$$\frac{1}{(1+3x)(1-x)} \equiv \frac{A(1-x)+B(1+3x)}{(1+3x)(1-x)}$$

Equate the numerators and substitute in values in order to get A, B:

$$1 = A(1-x)+B(1+3x) \quad \text{B1}$$

choose values to substitute in so that brackets become 0, to make your life easier!

$x=1 \Rightarrow 1 = A(0) + B(1+3(1))$

$1 = 4B \Rightarrow B = \frac{1}{4}$

$x=0 \Rightarrow 1 = A(1-0) + \frac{1}{4}(1+3(0))$

$1 = A + \frac{1}{4} \Rightarrow A = \frac{3}{4}$ M1

$$\therefore \frac{1}{(1+3x)(1-x)} \equiv \frac{3}{4(1+3x)} + \frac{1}{4(1-x)} \quad \text{A1}$$

Question 2 continued

(b)

$$(1+3x)(1-x)\frac{dy}{dx} = \tan y$$

$$\int \frac{1}{\tan y}\,dy = \int \frac{1}{(1+3x)(1-x)}\,dx$$

Separate variables by grouping the like terms on one side

$\frac{1}{\tan y} = \cot y$ ✓

$$\int \cot y\,dy = \int \frac{3}{4(1+3x)} + \frac{1}{4(1-x)}\,dx$$

Partial fractions from (a)

$$\Rightarrow \ln(\sin y) = \frac{3}{4}\ln(1+3x)\times\frac{1}{3} + \frac{1}{4}\ln(1-x)\times\frac{1}{-1}$$ M1M1

Reverse chain rule: divide by derivative of the bracket

$$\ln(\sin y) = \frac{1}{4}\ln(1+3x) - \frac{1}{4}\ln(1-x) + c$$ A1

$$\ln(\sin y) = \frac{1}{4}\ln\left(\frac{1+3x}{1-x}\right) + c$$

apply ln rule: $\ln a - \ln b = \ln\frac{a}{b}$

Get the value of +c using $y = \frac{\pi}{2}$, $x = \frac{1}{2}$:

$$\ln(\sin\tfrac{\pi}{2}) = \frac{1}{4}\ln\left(\frac{1+3\cdot\frac{1}{2}}{1-\frac{1}{2}}\right) + c$$

$$\ln(1) = \frac{1}{4}\left(\frac{\frac{5}{2}}{\frac{1}{2}}\right) + c$$

$$\Rightarrow c = -\frac{1}{4}\ln 5$$ dM1

Substitute our found +c value:

$$\ln(\sin y) = \frac{1}{4}\ln\left(\frac{1+3x}{1-x}\right) - \frac{1}{4}\ln 5$$

$$\ln(\sin y) = \frac{1}{4}\ln\left(\frac{1+3x}{5(1-x)}\right)$$

apply ln rule: $\ln a - \ln b = \ln\frac{a}{b}$

$$\cancel{\ln}(\sin y) = \cancel{\ln}\left(\frac{1+3x}{5(1-x)}\right)^{\frac{1}{4}}$$

apply ln rule: $a\ln b = \ln b^a$

$$\sin^4 y = \left(\left(\frac{1+3x}{5(1-x)}\right)^{\frac{1}{4}}\right)^4$$

Raise both sides to power of 4

$$\sin^4 y = \frac{1+3x}{5(1-x)}$$ M1A1

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3.

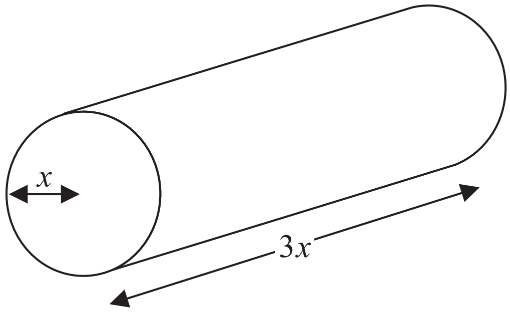

Figure 1

A tablet is dissolving in water.

The tablet is modelled as a cylinder, shown in Figure 1.

At t seconds after the tablet is dropped into the water, the radius of the tablet is x mm and the length of the tablet is $3x$ mm.

The cross-sectional area of the tablet is decreasing at a constant rate of $0.5 \text{ mm}^2 \text{ s}^{-1}$

(a) Find $\frac{dx}{dt}$ when $x = 7$ **(4)**

(b) Find, according to the model, the rate of decrease of the volume of the tablet when $x = 4$ **(4)**

(a) We are given that $\frac{dA}{dt} = -0.5 \text{mm}^2 \text{s}^{-1}$. B1

Cross section is just the area of a circle:

$$A = \pi x^2$$

$$\frac{dA}{dx} = 2\pi x \quad \text{B1}$$

$$\frac{dx}{dt} = \frac{dx}{dA} \times \frac{dA}{dt}$$

$$= \frac{1}{2\pi x} \times -0.5$$

$$\Rightarrow \frac{dx}{dt} = -\frac{1}{4\pi x} \quad \text{M1}$$

Substitute $x=7$:

$$\left.\frac{dx}{dt}\right|_{x=7} = -\frac{1}{28\pi}$$

$$\frac{dx}{dt} = -0.0113 \quad \text{A1}$$

P 7 1 3 8 1 A 0 8 3 2

Question 3 continued

(b) We are looking for the Rate of decrease of the volume, $\therefore \frac{dV}{dt}$

Volume of the cylinder:

$V = \pi r^2 l$

$$\therefore V = (\pi (x)^2) \cdot 3x$$
$$= 3\pi x^3 \quad \text{B1}$$

Get $\frac{dV}{dx}$:

$$\frac{dV}{dx} = 9\pi x^2 \quad \text{B1}$$

$$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} \text{ found in (a)}$$

$$= 9\cancel{\pi}x^2 \cdot -\frac{1}{4\cancel{\pi}x} \quad \text{M1}$$

$$= -\frac{9x^{\cancel{2}1}}{4\cancel{x}}$$

$$= -\frac{9}{4}x$$

Substitute $x = 4$:

$$\left.\frac{dV}{dt}\right|_{x=4} = -\frac{9}{4} \times 4 = -9$$

Hence Rate of decrease $= 9\ \text{mm}^3\text{s}^{-1}$ A1

(Total for Question 3 is 8 marks)

4. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$16x^3 - 9kx^2y + 8y^3 = 875$$

where k is a constant.

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2}$$

(4)

Given that the curve has a turning point at $x = \frac{5}{2}$

(b) find the value of k

(4)

(a) ✯ Implicit differentiation:

When differentiating y, multiply by $\frac{dy}{dx}$

Product Rule: $\frac{d}{dx}(uv) = uv' + u'v$

$$16x^3 - 9kx^2y + 8y^3 = 875$$

Product Rule more clearly:

$u = 9kx^2 \quad u' = 18kx$

$v = y \quad v' = \frac{dy}{dx}$

Differentiate:

$$48x^2 - 9kx^2\frac{dy}{dx} - 18kxy + 24y^2\frac{dy}{dx} = 0$$

B1M1

$$48x^2 - 18kxy = 9kx^2\frac{dy}{dx} - 24y^2\frac{dy}{dx}$$

Get all $\frac{dy}{dx}$ on one side

$$48x^2 - 18kxy = \frac{dy}{dx}(9kx^2 - 24y^2)$$

Factor out $\frac{dy}{dx}$

$$\Rightarrow \frac{dy}{dx} = \frac{48x^2 - 18kxy}{9kx^2 - 24y^2} \begin{matrix}\times -1 \\ \times -1\end{matrix} = \frac{18kxy - 48x^2}{24y^2 - 9kx^2}$$

Multiply up and down by -1

M1

$$\frac{dy}{dx} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2}$$

A1

Divide up and down by 3

hence shown

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Question 4 continued

(b) At the turning point, $\frac{dy}{dx}=0$.

$$\frac{dy}{dx} = \frac{6k\left(\frac{5}{2}\right)y-16\left(\frac{5}{2}\right)^2}{8y^2-3k\left(\frac{5}{2}\right)^2}=0 \quad \text{M1}$$

$$\Rightarrow 15ky-100=0 \quad \text{A1}$$

$$y=\frac{20}{3k}$$

Now use the given $x=\frac{5}{2}$ and initial equation with substitution $y=\frac{20}{3k}$ to get k:

$$16\left(\frac{5}{2}\right)^3-9k\left(\frac{5}{2}\right)^2\left(\frac{20}{3k}\right)+8\left(\frac{20}{3k}\right)^3=875 \quad \text{M1}$$

$$16\cdot\left(\frac{125}{8}\right)-\cancel{9}^{3}\cancel{k}\left(\frac{25}{\cancel{4}_1}\right)\left(\frac{\cancel{20}^{5}}{\cancel{3k}_1}\right)+8\left(\frac{8000}{27k^3}\right)=875$$

$$250-375+\frac{64000}{27k^3}=875$$

$$\frac{64000}{27k^3}=1000$$

$$64=27k^3$$

$$k^3=\frac{64}{27} \Rightarrow k=\frac{4}{3} \quad \text{A1}$$

Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Use the substitution $x = 2\sin u$ to show that

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,dx = \int_0^p \left(\frac{3}{2}\sec u \tan u + \frac{1}{2}\sec^2 u\right) du$$

where p is a constant to be found.

(4)

(b) Hence find the exact value of

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,dx$$

(4)

P 7 1 3 8 1 A 0 1 4 3 2

Question 5 continued

(a) To use the given substitution $x = 2\sin u$, we need to find new limits and $dx = \ldots$

New Limits:

x		u
0	$0 = 2\sin u \rightarrow$	0
1	$\frac{1}{2} = \sin u \rightarrow$	$\frac{\pi}{6}$ $(=p)$

B1

To get dx, differentiate $x = 2\sin u$:

$$x = 2\sin u$$
$$\frac{dx}{du} = 2\cos u$$
$$dx = 2\cos u\, du$$

M1

Substitute into the Integration:

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,dx$$

$$= \int_0^{\frac{\pi}{6}} \frac{3(2\sin u)+2}{(4-(2\sin u)^2)^{\frac{3}{2}}} \cdot 2\cos u\,du$$ M1

$$= \int_0^{\frac{\pi}{6}} \frac{(6\sin u+2)\cdot 2\cos u}{(4-4\sin^2 u)^{\frac{3}{2}}}\,du$$ expand up & down

$$= \int_0^{\frac{\pi}{6}} \frac{12\sin u\cos u+4\cos u}{(4)^{\frac{3}{2}}(1-\sin^2 u)^{\frac{3}{2}}}\,du$$ expand up, factor 4 out down

$$= \int_0^{\frac{\pi}{6}} \frac{4\cos u(3\sin u+1)}{8(\cos^2 u)^{\frac{3}{2}}}\,du$$ factor $\cos u$ out up, use Identity $\cos^2 x + \sin^2 x = 1$ to replace $(1-\sin^2 u)$

$(\cos^2 u)^{\frac{3}{2}} = (\cos^2 u)^{\frac{1}{2}} \cdot (\cos^2 u)^1 = \cos u(\cos^2 u)$

$$= \int_0^{\frac{\pi}{6}} \frac{{}^1\cancel{4}\cancel{\cos u}(3\sin u+1)}{{}_2\cancel{8}\cancel{\cos u}(\cos^2 u)}\,du$$

$$= \int_0^{\frac{\pi}{6}} \frac{3\sin u+1}{2\cos^2 u}\,du$$

$$= \int_0^{\frac{\pi}{6}} \frac{3\sin u}{2\cos^2 u} + \frac{1}{2\cos^2 u}\,du$$ Separate into two fractions

$$= \int_0^{\frac{\pi}{6}} \frac{3}{2}(\tan u)\cdot\left(\frac{1}{\cos u}\right) + \frac{1}{2}\left(\frac{1}{\cos^2 u}\right)du$$ use $\frac{\sin u}{\cos u} = \tan u$

$$= \int_0^{\frac{\pi}{6}} \frac{3}{2}\tan u\sec u + \frac{1}{2}\sec^2 u\,du$$ use $\frac{1}{\cos u} = \sec u$

A1 hence shown

P 7 1 3 8 1 A 0 1 5 3 2

Question 5 continued

(b) $\int_0^{\frac{\pi}{6}} \frac{3}{2}\tan u \sec u + \frac{1}{2}\sec^2 u \, du$ Standard Integral $\int \sec^2 x \, dx = \tan x + c$

Integration By Recognition, in this case $\int f'(x)\,dx$ since we know that $\tan x \sec x = \frac{d}{dx}(\sec x)$.

$= \left[\frac{3}{2}\sec u + \frac{1}{2}\tan u\right]_0^{\frac{\pi}{6}}$ M1A1

$= \left(\frac{3}{2}\sec\frac{\pi}{6} + \frac{1}{2}\tan\frac{\pi}{6}\right) - \left(\frac{3}{2}\sec(0) + \frac{1}{2}\tan(0)\right)$ M1

$= \frac{3}{2}\cdot\frac{2\sqrt{3}}{3} + \frac{1}{2}\cdot\frac{\sqrt{3}}{3} - \frac{3}{2}$

$= \sqrt{3} + \frac{\sqrt{3}}{6} - \frac{3}{2}$

$= \frac{7\sqrt{3}}{6} - \frac{3}{2}$ exact value

A1

Question 5 continued

(Total for Question 5 is 8 marks)

6. Relative to a fixed origin O,

- the point A has position vector $\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$
- the point B has position vector $5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$
- the point C has position vector $3\mathbf{i} + p\mathbf{j} - \mathbf{k}$

where p is a constant.

The line l passes through A and B.

(a) Find a vector equation for the line l

(3)

Given that $\overrightarrow{AC}$ is perpendicular to l

(b) find the value of p

(3)

(c) Hence find the area of triangle ABC, giving your answer as a surd in simplest form.

(3)

(a) Formula for vector line:

r = position vector + λ(direction vector)

Where the position vector is any point on the line and direction vector is a vector parallel to the line.

Get direction vector:

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix} \quad \text{M1}$$

Hence get vector line in the format $r = \overrightarrow{OA} + \lambda \overrightarrow{AB}$

$$r = \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix} - \lambda \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix} \quad \text{M1A1}$$

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Question 6 continued

(b) Let's get $\vec{AC}$ in terms of p:

$\vec{AC} = \vec{OC} - \vec{OA}$

$= \begin{pmatrix} 3 \\ p \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ p+4 \\ -4 \end{pmatrix}$ M1

Formula for dot product:

$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix} = ad + be + cf$

Since l_1 and $\vec{AC}$ are given to be perpendicular, the dot product of $\vec{AC}$ and l_1's direction vector is 0.

$$\therefore \vec{AB} \cdot \vec{AC} = 0$$

$$\begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ p+4 \\ -4 \end{pmatrix} = 0$$

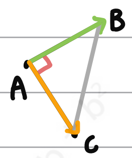
$$8 + 7(p+4) + 20 = 0$$ Solve for p

$$8 + 7p + 28 + 20 = 0$$

$$7p = -56$$

$$p = -8$$ M1A1

(c) Diagram



We know that $\vec{AB}$ is perpendicular to $\vec{AC}$, so the area of triangle ABC is just:

$$\text{area} = \frac{|\vec{AB}||\vec{AC}|}{2}$$

Get magnitudes:

$\vec{AB} = \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix}$ $\quad |\vec{AB}| = \sqrt{4^2+7^2+(-5)^2} = \sqrt{90}$

$\vec{AC} = \begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix}$ $\quad |\vec{AC}| = \sqrt{2^2+(-4)^2+(-4)^2} = \sqrt{6}$ M1

$$\therefore \text{area} = \frac{\sqrt{90} \times \sqrt{6}}{2}$$ dM1

$$= 9\sqrt{10}$$ A1

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has parametric equations

$$x = \sin t - 3\cos^2 t \qquad y = 3\sin t + 2\cos t \qquad 0 \leqslant t \leqslant 5$$

(a) Show that $\dfrac{dy}{dx} = 3$ where $t = \pi$

(4)

The point P lies on C where $t = \pi$

(b) Find the equation of the tangent to the curve at P in the form $y = mx + c$ where m and c are constants to be found.

(3)

Given that the tangent to the curve at P cuts C at the point Q

(c) show that the value of t at point Q satisfies the equation

$$9\cos^2 t + 2\cos t - 7 = 0$$

(2)

(d) Hence find the exact value of the y coordinate of Q

(3)

(a) ★ Parametric differentiation:

$\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$ → get these by differentiating the parametrics separately.

Get $\dfrac{dy}{dt}$ and $\dfrac{dx}{dt}$

$y = 3\sin t + 2\cos t$

$\dfrac{dy}{dt} = 3\cos t - 2\sin t$ B1

$x = \sin t - 3\cos^2 t$

$\dfrac{dx}{dt} = \cos t - 6\cos t(-\sin t)$ chain rule: multiply by derivative of the bracket

$\dfrac{dx}{dt} = \cos t + 6\sin t\cos t$ B1

Hence get $\dfrac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{dy}{dx} = \frac{3\cos t - 2\sin t}{\cos t + 6\sin t\cos t} \quad \text{M1}$$

Substitute $t = \pi$:

$$= \frac{3\cos\pi - 2\cancel{\sin\pi}^{\,0}}{\cos\pi + 6\cancel{\sin\pi}^{\,0}\cos\pi}$$

$$= \frac{-3}{-1} = 3$$ hence shown

A1

Question 7 continued

(b) We know $\frac{dy}{dx}=3$ at P. (given and proven in (a)).

Get the cartesian point P:

$y=3\cancel{\sin\pi}^{0}+2\cos\pi$

$y=-2$

$x=\cancel{\sin\pi}^{0}-3\cos^2\pi$

$x=-3$ B1

Use formula $y-y_1=m(x-x_1)$ to get the equation of the line:

$y-(-2)=3(x-(-3))$ M1

$y+2=3(x+3)$

$\boxed{y=3x+7}$ A1

(c) To get that equation we need to substitute the component equations into our found tangent.

$y=3x+7$ tangent equation

M1 $3\sin t+2\cos t=3(\sin t-3\cos^2 t)+7$ Substitute in $y=\ldots$ and $x=\ldots$

$\cancel{3\sin t}+2\cos t=\cancel{3\sin t}-9\cos^2 t+7$ expand

A1 $\boxed{0=9\cos^2 t+2\cos t-7}$ Rearrange

hence shown

(d) Solve $9\cos^2 t+2\cos t-7=0$ for t:

$(9\cos t-7)(\cos t+1)=0$

$\cos t=\frac{7}{9}$ $\quad$ $\cos t=-1$

B1 $\quad$ $t=\pi$ (Point P)

Hence at Point Q, $\cos t=\frac{7}{9}$.

Get $\sin t$:

$\sin^2 t+\cos^2 t=1$

$\sin^2 t=1-\left(\frac{7}{9}\right)^2$

$\sin t=\sqrt{1-\frac{49}{81}}$

$=\sqrt{\frac{32}{81}}=\frac{\sqrt{32}}{9}=\sin t$

Substitute this all into $y=\ldots$:

$y=3\sin t+2\cos t$

$y=\cancel{3}\left(\frac{\sqrt{32}}{\cancel{9}_{3}}\right)+2\left(\frac{7}{9}\right)$

$=\frac{\sqrt{32}}{3}+\frac{14}{9}=\frac{4\sqrt{2}}{3}+\frac{14}{9}$

$\boxed{y=\frac{12\sqrt{2}+14}{9}}$ y-coord. of Q

M1A1

Question 7 continued

P 7 1 3 8 1 A 0 2 4 3 2

Question 7 continued

(Total for Question 7 is 12 marks)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

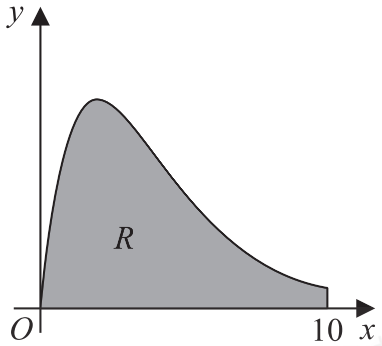


Figure 2

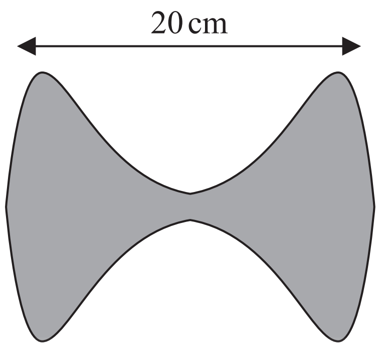


Figure 3

Figure 2 shows the curve with equation

$$y = 10xe^{-\frac{1}{2}x} \qquad 0 \leqslant x \leqslant 10$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line with equation $x = 10$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the volume, V, of this solid is given by

$$V = k\int_0^{10} x^2 e^{-x}\,dx$$

where k is a constant to be found.

(2)

(b) Find $\int x^2 e^{-x}\,dx$

(3)

Figure 3 represents an exercise weight formed by joining two of these solids together.

The exercise weight has mass 5 kg and is 20 cm long.

Given that

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

and using your answers to part (a) and part (b),

(c) find the density of this exercise weight. Give your answer in grams per cm^3 to 3 significant figures.

(5)

P 7 1 3 8 1 A 0 2 6 3 2

Question 8 continued

(a) Formula for Volume of Revolution around the x-axis:

$$V = \pi \int_a^b y^2 \, dx$$

where a and b are the points on the x-axis bounding the area

$$V = \pi \int_0^{10} \left(10xe^{-\frac{1}{2}x}\right)^2 dx$$

$$= \pi \int_0^{10} 100x^2 e^{-x} \, dx$$

$$= 100\pi \int_0^{10} x^2 e^{-x} \, dx$$ hence shown

(b) $$\int x^2 e^{-x} \, dx$$ multiplication $\therefore$ use By Parts

★ Integration by Parts:

$$\int uv' \, dx = uv - \int u'v \, dx$$

$u = x^2 \xrightarrow{\frac{d}{dx}} u' = 2x$

$v = -e^{-x} \xleftarrow{\int} v' = e^{-x}$

$$= -x^2 e^{-x} - \int -2xe^{-x} \, dx$$

$$= -x^2 e^{-x} + \int 2xe^{-x} \, dx$$ Still a multiplication $\therefore$ By Parts again

$u = 2x \xrightarrow{\frac{d}{dx}} u' = 2$

$v = -e^{-x} \xleftarrow{\int} v' = e^{-x}$

$$= -x^2 e^{-x} - 2xe^{-x} - \int -2e^{-x} \, dx$$ Finally Integrate

$$= -x^2 e^{-x} - 2xe^{-x} - 2e^{-x} + c$$

Question 8 continued

(c) Let's get the total volume of the solid:

$$V = 2 \cdot 100\pi \int_0^{10} x^2 e^{-x}\,dx$$

$$= 200\pi \left[-x^2e^{-x} - 2xe^{-x} - 2e^{-x} \right]_0^{10}$$

$$= 200\pi \left(\left(-(10)^2e^{-10} - 2(10)e^{-10} - 2e^{-10} \right) - \left(-(0)^2e^{0} - 2(0)e^{0} - 2e^{0} \right) \right)$$

$$= 200\pi \left(-100e^{-10} - 20e^{-10} - 2e^{-10} + 2 \right)$$

$$= 200\pi \left(2 - 122e^{-10} \right)$$ Volume

Use the given density formula:

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{density} = \frac{5000}{200\pi(2-122e^{-10})}$$

given, 5kg → 5000g

found above

$\therefore$ density $= 3.99\,\text{gcm}^{-3}$

Question 8 continued

(Total for Question 8 is 10 marks)

9. Use proof by contradiction to show that, when n is an integer,

$$n^2 - 2$$

is **never** divisible by 4

(4)

Make an Assumption:

"Assume that there is a real number n for which n^2-2 is divisible by 4" M1

We want our assumption to be the opposite of what we are trying to prove.
In the proof, we are trying to contradict the assumption

Hence:

$n^2 - 2 = 4k$

$\Rightarrow n^2 = 4k + 2$

$n^2 = 2(2k+2)$ Since it's a multiple of 2, n^2 is even A1

Since n^2 is even, n must be even $\therefore n = 2m$.

$n^2 - 2 = (2m)^2 - 2$

$= 4m^2 - 2$

$n^2 - 2 = 2(2m^2 - 1)$ $2m^2 - 1$ is odd, hence $n^2 - 2$ is odd

dM1

If $n^2 - 2$ is odd it can't be a multiple of 4, which is a contradiction to our assumption. Hence it's proven by contradiction that "$n^2 - 2$ is never divisible by 4" is true for all n.

A1

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DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

P 7 1 3 8 1 A 0 3 0 3 2

Question 9 continued

Question 9 continued

(Total for Question 9 is 4 marks)

TOTAL FOR PAPER IS 75 MARKS